ME-B41 Lab 1: Hydrostatics

In this lab you will do four brief experiments related to the following topics: manometry, buoyancy, forces on submerged planes, and hydraulics (a hydraulic jack). Each experiment should take less than 15 minutes and the write-up should take 1 hour or less for each.

Experimental Procedures

Manometry:

WHAT YOU ARE GOING TO DO: You will find the density of the fluid in one manometer by measuring the pressure applied to that manometer using a second manometer with a fluid of known density.

WHAT YOU SHOULD LEARN: You should learn how a manometer works and how the elevation in a manometer depends on the density of the fluid in the manometer.

DETAILS: In this experiment there are two U-tube manometers. One leg of each manometer is connected by a flexible tube to a hand pump. The purple fluid in one manometer has a known density of $\rho=1$ gm/cm^3. The flexible tube connecting the manometers is filled with air. The red fluid in the second manometer has an unknown density. You are to find the density of the red fluid.

Procedure:

p1. Sketch the set-up on the back of the attached data sheet. (This is for your reference when you write the lab report).

p2. Apply a vacuum to the tubes between the manometers using the hand pump. Be careful: a short ***gentle*** stroke on the pump is very effective. Do not overflow the manometer.

p3. Measure the difference in height between the two legs of each U-tube (both purple and red) and record on the data sheet. Release the vacuum using the red lever on the hand pump nozzle.

p4. Repeat steps p2 and p3 three more times, each time with a different applied vacuum, to obtain a total of four readings.

Calculations:

c1. Use the equation for hydrostatic pressure of a constant density fluid to write an expression for the difference between air pressure in the tube between manometers and atmospheric pressure using the difference in height between the two legs of U-tube for the purple fluid; repeat for red fluid.

c2. Set the equations in c1 equal.

c3. Use equation in c2 to find density of the red fluid for the first applied vacuum; repeat for all readings.

c4. Calculate the average density of the red fluid found from the four measurements.

Write-Up: Include the following in your write-up:

1. Derivation of equations in steps c1 and c2; Sample of calculation (step c3).
2. Table of red fluid density for each applied pressure. Also include the average density in the table.
3. Discuss briefly (a few sentences per topic)
    - Why can the effect of the air in the tube between the two manometers be neglected?
    - Sources of error.
4. Manometry Data Sheet

MANOMETRY DATA SHEET

	Run #1	Run #2	Run #3	Run #4
Δh† Purple liquid (ρ = 1.000 g/cm^3)	in.	in.	in.	in.
Δh Red liquid	in.	in.	in.	in.

† Δh signifies height difference

Note any apparant sources of error in your measurements:

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Buoyancy:

WHAT YOU ARE GOING TO DO: You will find the density (mass/volume) of two different solid bodies in two different ways: 1) measure the volume of the body directly; 2) measure the volume using buoyancy relations.

WHAT YOU SHOULD LEARN: You should learn how to get an accurate measure of a solid's volume using buoyancy.

DETAILS: In this experiment, you are to find the density two different cylindrical bodies--one solid metal and one hollow plastic.

Procedure:

p1. You may ignore the weight of the cord, rod, hook, etc. in this experiment.

p2. Sketch the metal body (body a) and measure its dimensions to the nearest 0.1 cm.

p3. Screw the small hook all of the way into the rod at the bottom of the scale until the hook bottoms out. Zero the scale using the thumbwheel below the display. Weigh the body by hanging it from the scale, so that it is not submerged in the water. Be careful not to drop the weight in the water or torque the shaft connected to the scale. Record the weight.

p4. Use the cord between the body and the scale so that the body is ***fully immersed*** in the water. Record the weight of the body when it is fully immersed (convert to N). This is equivalent to the tension force in the cord.

p5. Sketch the experimental setup on the back of your data sheet. (This is for your reference when you write the lab report).

p6. Repeat steps p2 through p5 for the plastic hollow body (body b), except here you must use the a rigid rod between the body and the scale to keep the body fully immersed in the water in step p4. The rigid rod is needed because the body floats, and you need to record the net upward force of the body on the scale when the body is submerged. To do this, screw the threaded rod into the plastic hollow body. Remove the hook from the rod attached to the scale and slide the narrow end of the rod attached to the plastic body into the rod attached to the scale. ***(Use caution so that the rod does not bend.)***

Calculations:

c1. Using the dimensions that you measured in 1 above, calculate the volume of the body a in cm^3.

c2. Calculate the density of the body using the weight of the body and the volume from step c1, in gm/cm^3.

c3. Draw a free body diagram of the body when it is fully submerged in the fluid. Include the weight force, buoyancy force, and tension for in the string to the scale.

c4. Using the principles of buoyancy and a force balance on the body, calculate the volume of the body in cm^3.

c5. Calculate the density of the body using the volume from step c4, in gm/cm^3.

c6. Repeat calculations for body b.

Write-Up: Include the following in your write-up:

1. Calculations of volume and density (c1 and c2) based on dimensions for each body.
2. Free Body Diagram of each body and calculations of volume and density (c3, c4, and c5) for each body.
3. Discuss briefly (a few sentences per topic)
    - Which method for calculating density is more accurate? Why?
    - Sources of error.
4. Buoyancy Data Sheet

Note: 1 lbf = 4.4482 N = 444,820 $gm\text{-}cm/s^2$

BUOYANCY DATA SHEET

d_a = __________ cm (diameter of body a)

h_a = __________ cm (height of body a)

W_a^{air} = __________ lbf = ___________ N (weight of body a in air)

W_a^{liquid} = __________ lbf = __________ N (weight of body a in liquid)

d_b = __________ cm (diameter of body b)

h_b = __________ cm (height of body b)

W_b^{air} = __________ lbf = ___________ N (weight of body b in air)

W_b^{liquid} = __________ lbf = ___________ N (weight of body b in liquid)

Note any apparant sources of error in your measurements:

Force on a submerged plane:

WHAT YOU ARE GOING TO DO: You will measure the force exerted by water on a vertical plane and compare this force to the calculated force.

WHAT YOU SHOULD LEARN: You should learn how to calculate forces on submerged planes.

DETAILS:
In this experiment, you are to measure the force that water in a tank exerts on a vertical planar surface that forms one of its side walls. (Be careful, you cannot directly measure the force. But you can measure the force to hold the side wall in place and then calculate the force of the fluid based on summing the moments of the forces acting on the side wall.) Then you will calculate the force acting on the side wall based on hydrostatics.

Procedure:

p1. Sketch the experimental setup on the back of the data sheet. (This is for your reference when you write the lab report).
  a . Note on your sketch the position of the hinge at the bottom of the side wall and where the water is.
  b. The side wall plate is constructed from two plates with a rubber gasket between them. Measure the size of the side wall plate (length, width, and total thickness of both plates including the rubber gasket) to the nearest 1/16 in.
  c. Measure the distance from the hinge to the surface of the water.
  d. Measure the distance from the hinge to the eyebolt where the cord is attached.

p2. Carefully remove the wooden block supporting the vertical plate. If necessary, gently adjust the turnbuckle on the cable to make the plate vertical.

p3. Gently push the vertical plate toward the force gauge so that there is no tension. Zero the gage using the button near the 4 lb mark and then slowly release the vertical plate so that the cable tightens. Record the the tension in the cable by reading on the force gauge. (This gage only reads the maximum force so it is crucial to release the vertical plate slowly).

p4. Replace the block supporting the hinged door.

Calculations:

c1. Using the dimensions that you measured in p1a and p1b, calculate the force of the water on the hinged plate. Also calculate the vertical location, measured from the hinge, where the resultant force of the water acts on the vertical plate. Remember that the area used in these calculations is the wetted area of the vertical plate, not the total area of the vertical plate.

c2. Using the dimensions of the hinged door measured in p1b, calculate the weight of the door. The specific gravity of the acylic (plastic) door is 1.25.

c3. Draw a free body diagram of the forces acting on the plate (T=tension in cable, F=force of water, W=weight of plate, R_x & R_y=hinge reaction forces). Neglect the force of the rubber gasket material at the side and bottom edges of the side wall plate. Note that W acts the center of gravity of the plate, so that it exerts a moment about the hinge.

c4. Assume that the tension in the cable is unknown and use the other forces in the free body diagram to calculate the tension by summing the moments about the hinge.

Write-Up: Include the following in your write-up:

1. Calculations of force of the water on the side wall plate and its line of action (c1).
2. Calculation of the weight of the plate (c2).
3. Free Body Diagram of the side wall plate (c3).
4. Calculation of cable tension (c4).
5. Discuss briefly (a few sentences per topic)
   • Compare the tension in the cable measured directly (p3) with that calculated based on the force of the water (c3). If they are differenct, what are the sources of error?
6. Force on a Submerged Plane Data Sheet

Force on a Submerged Plane Data Sheet

t = ___________ in (door thickness)

h = ___________ in (door height)

w = ___________ in (door width)

d = ___________ in (vertical distance from point of rotation to liquid/air interface)

l = ___________ in (vertical distance from point of rotation to attachment site of resisting cable)

T = ___________ lbf (tension in resisting cable)

Note any apparant sources of error in your measurements:

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Practical Application--Hydraulic Jack:

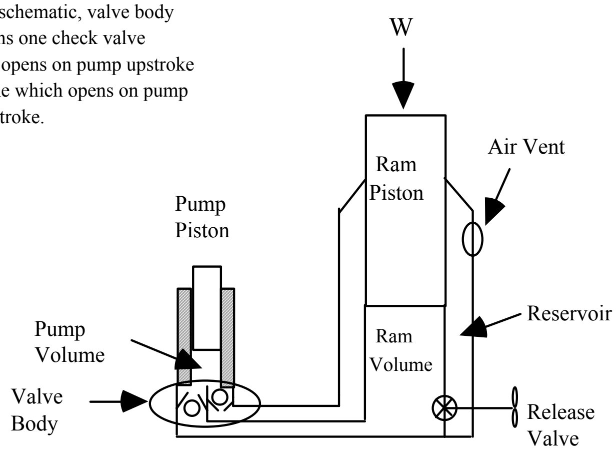
WHAT YOU ARE GOING TO DO: You will take apart a hydraulic jack and determine the work done by the jack in lifting a weight.

WHAT YOU SHOULD LEARN: You should learn how a hydraulic jack is used to obtain leverage.

DETAILS: The workings of common instruments are not always intuitive. For instance, a hydraulic jack lifts very heavy objects without requiring a lot of human effort to operate it. This is because a jack incorporates leverage, as do similar tools such as a block and tackle, a winch, and a prybar. In leveraging effort, the application of a small force through a long distance has the same effect as a greater force applied through a shorter distance.

There are three identical hydraulic jacks on display for this lab: 1) jack with no fluid so that you can take it apart to see how it works; 2) jack with parts of it cut away to expose its inner workings; 3) jack that works and is used to make measurements. Below is a schematic of the pump.

In the schematic, valve body contains one check valve which opens on pump upstroke and one which opens on pump downstroke.



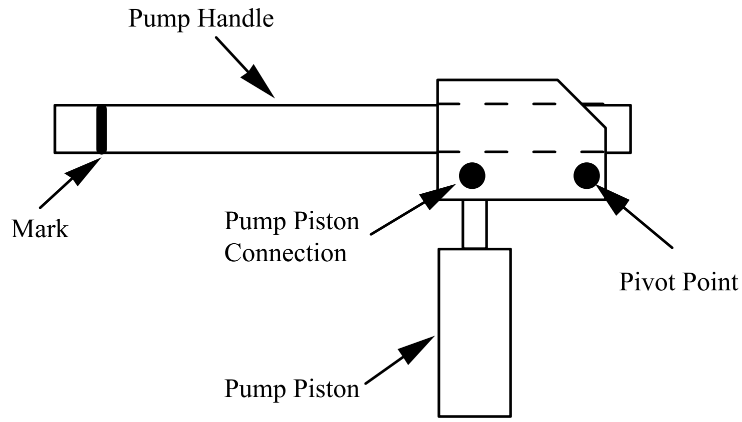
Note the following:

* Assume all three jacks in this lab are identical in every respect, so that you can make measurements on one jack and apply them to another jack.
* Remember that energy is conserved (it is not created or destroyed, although it can be transformed from one type to another).
* Work = $W = \int \mathbf{F} \cdot ds$, where **F** is the vector force and **s** is the vector displacement. Note that for a constant force parallel to the displacement we can use the magnitudes and $W = F \int ds = Fs$.
* Assume that the fluid used in the jack is incompressible and isothermal (remains at the same temperature) throughout the exercise. In other words, assume that all of your effort applied to the jack handle goes into raising the weight and doesn't go into raising the temperature of the fluid via compression or friction.
* The jack's "leverage" is determined by the relative sizes of the ram (large) cylinder diameter and the pump (small) cylinder diameter. These will determine the ***distance*** each piston ***travels*** in the ram and pump cylinders.

Procedure:

p1. Take apart the fluid-less model and measure:

- The distance from the mark on the handle to the pivot point. (Measure the distance parallel to the pump handle.)
- The distance from the pump piston connection point to the pivot point.
- The inside diameters of both the ram cylinder and the pump cylinder (not the piston diameter).



Pump Piston and Handle Assembly

p2. Figure out how the hydraulic jack works. Note the function of the valves.

p3. Lift the weight by pumping the handle several times. Make sure that the weight is high enough so that it does not interfere with a complete stroke of the handle. Record the weight including the weight of the platform and bar (11.1 lbf).

p4. Measure the distance traveled by the pump piston in one stroke of the handle by standing a ruler up next to the pump and noting how far the wire marker moves during a pump stroke.

p5. Measure the distance traveled by the ram piston (ram plunger) in three pumps of the handle by standing a ruler up next to the pump and noting how far the wire marker moves during the pump strokes.

Calculations:

c1. Sketch a free body diagram of the ram piston.

c2. Calculate the pressure in the hydraulic fluid in the chamber between the pump piston and the ram piston using the free body diagram of the ram piston.

c3. Sketch a free body diagram of the pump piston and handle assembly (including the pivot point and the pump piston connection).

c4. Calculate the minimum force that needs to be applied to the mark on the handle for the given weight based on the free body diagram in c3 by summing the moments about the pivot point.

c5. Calculate the work done by the pump piston in one stroke of the handle.

c6. Calculate the work done by the ram piston in one stroke of the handle. (You measured the travel for three strokes of the handle so you need to divide this measurement by 3 before calculating the work).

Write-Up: Include the following in your write-up:

1. Calculation of the pressure in the hydraulic fluid (c1 and c2).
2. Calculation of the minimum force applied at the mark on the handle to lift the weight (c3 and c4).
3. Calculation of the work done by the pump piston and the ram piston (c5 and c6).
4. Discuss briefly (a few sentences per topic)
   - Describe how the hydraulic jack works. What is the function of each of the three valves?
   - Why is the work done by both pistons the same?
   - In an analysis of the hydraulic jack we neglected friction due to o-rings, hinges, etc. Why can we do this?

5. Hydraulic Jack Data Sheet

HYDRAULIC JACK DATA SHEET

r_h=____________________ in. distance from mark on handle to pivot pt.

r_p=____________________ in. distance from pump piston connection to pivot pt.

d_r=____________________ in. diameter of ram cylinder (not the piston)

d_p=____________________ in. diameter of pump cylinder (not the piston)

W=____________________ lbf. weight including weight platform and bar

$dist_{pump}$=____________________ in. distance traveled by pump(1 stroke of pump)

$dist_{ram}$=_____________________ in. distance traveled by ram(3 strokes of pump)

Note any apparant sources of error in your measurements:

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Appendix: Fluid Statics

When a fluid has no motion at all, the pressure gradient in the fluid balances the body forces in the fluid. This can be expressed mathematically as

$$p = \rho \mathbf{f} \qquad (1),$$

where is the vector gradient operator, p is the pressure, ρ is the fluid density, and **f** is the vector body force. The most common body force is gravity. If z is positive upward from the surface of the earth, then the body force **f** is the acceleration of gravity, g, in the negative z direction. The body force is zero in the x and y direction. Equation (1) becomes

$$\frac{dp}{dz} = -\rho g \qquad (2).$$

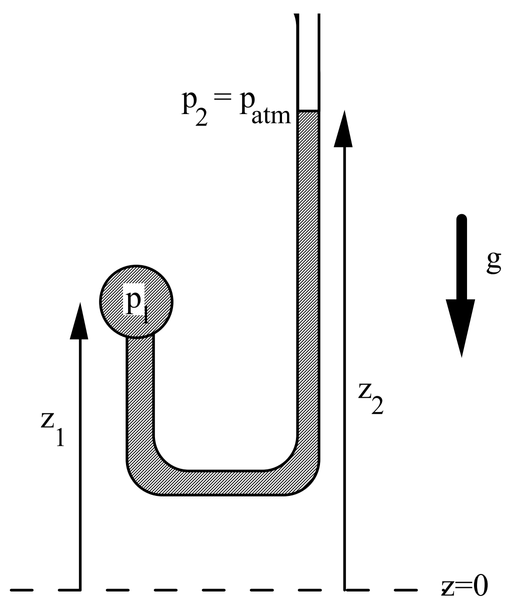
The minus sign on the right hand side results from the acceleration of gravity in the negative z direction.

If the density and acceleration of gravity are independent of z (which they usually are in a laboratory situation), equation (2) can be integrated to

$$p_2 - p_1 = -\rho g (z_2 - z_1) \qquad (3).$$

Manometry:

From equation (3) it is apparent that the height of a static fluid column can be used to measure a pressure difference, if the density of the fluid is known. This is the basis of a device known as a manometer. In a manometer, the difference in height of fluid in the legs of a U-tube indicate the difference in pressure applied to each of the two legs. Consider the example shown below.



The left leg of the manometer is exposed to an unknown pressure p_1. The pressure in the right leg is atmospheric, so $p_2 = p_{atm}$. The coordinates z_1 and z_2 are measured from an arbitrary reference plane. Substituting into equation (3) results in

$$p_1 = \rho g (z_2 - z_1) + p_{atm}$$

Multiple fluids in a manometer are handled similarly by careful and consistent application of equation (3).

Buoyancy:

Two principles, developed by Archimedes in about 300 B.C., describe buoyancy forces on submerged or partially submerged bodies. These two principles are:

1) The buoyant force acting on a body, F_b, equals the weight of the fluid displaced by a body. This can be written as

$$F_b = \rho\, g\, V_{displaced} \quad (4),$$

where $V_{displaced}$ is the volume of fluid displaced by the body.

2) A floating body displaces its own weight in fluid. This is written as

$$W = \rho\, g\, V_{displaced} \quad (5),$$

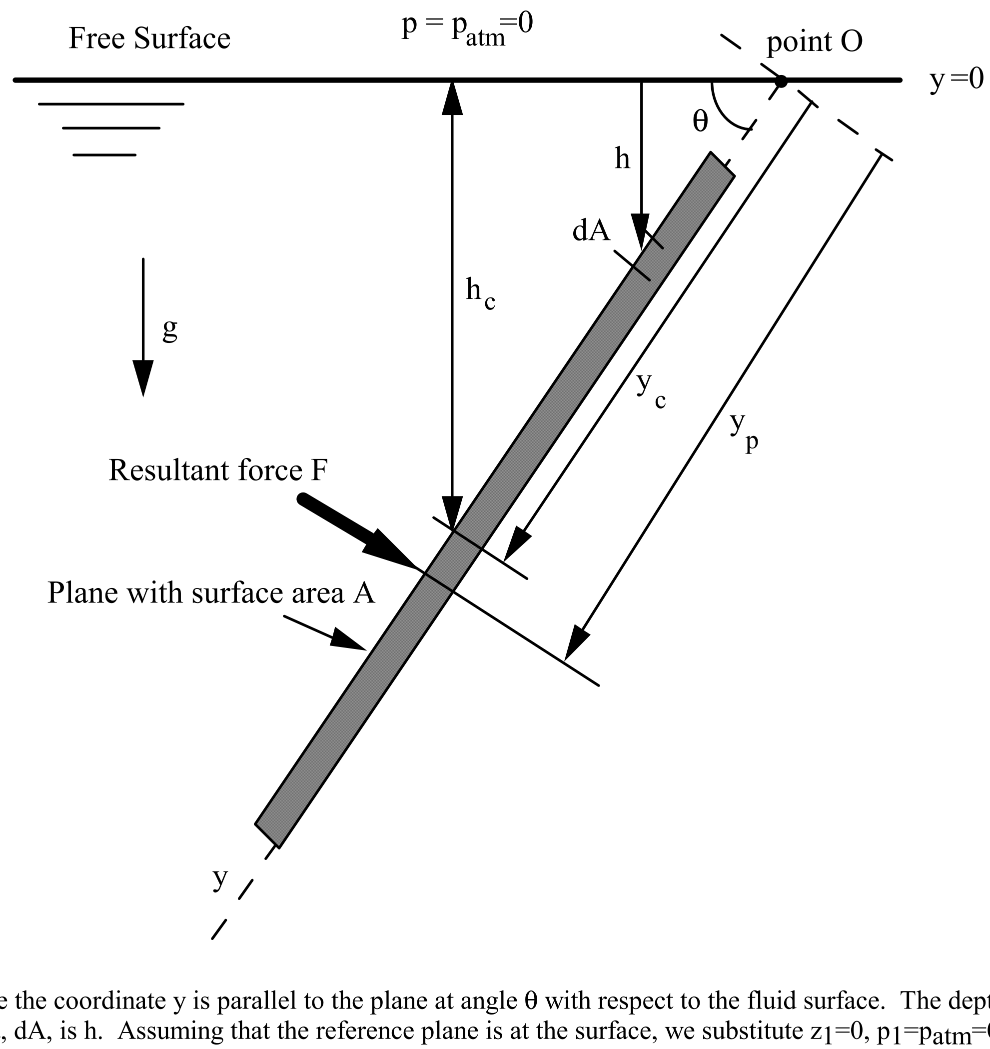
where W is the weight of the body.

Forces on submerged plane surfaces:

The fluid static pressure will exert a resultant force on a submerged surface that is given by

$$F = \int_A p\, dA \quad (6),$$

where A is the "wetted" area over which the pressure acts. Consider the submerged plane surface shown below.



Here the coordinate y is parallel to the plane at angle θ with respect to the fluid surface. The depth to an incremental area, dA, is h. Assuming that the reference plane is at the surface, we substitute z_1=0, p_1=p_{atm}=0, and z_2=-h=(y sin θ) into equation (3) to obtain the pressure, p, acting on dA at any arbitrary z_2 on the surface as

$$p = \rho\, g\, y \sin\theta \quad (7).$$

Substituting (7) into (6) yields

$$F = \rho\ g \left[\sin\theta \int_A y\ dA \right] \qquad (8).$$

But the integral is simply the *centroid* of the surface times the area of the surface, y_c A. The quantity in brackets is the *depth of the centroid* of the surface, h_c, times the area of the surface, since $h_c A = (y_{cent} \sin\theta)\ A$. Now (8) can be rewritten as

$$F = \rho\ g\ h_c\ A \qquad (9).$$

Thus, all that is needed to calculate the force on the submerged plane surface is the area of the surface that is in contact with the fluid, A, and the distance from the fluid surface to the centroid of the surface, h_c.

Now the question that remains is where does the resultant force, F, act on the submerged surface? This can be answered by considering the moment balance about point O, where y_p is the distance parallel to the surface to the center of pressure where F acts. The moment balance is

$$y_p F = \int_A y\ dF \qquad (10).$$

Noting that dF = p dA and using (7) and (9), equation (10) can be rewritten as

$$y_p\ (\rho\ g\ h_c\ A) = \int_A y\ (\rho\ g\ y \sin\theta)\ dA \qquad (11).$$

Simplifying results in

$$y_p\ h_c\ A = \sin\theta \int_A y^2\ dA \qquad (12).$$

But the integral is simply the moment of inertia about the horizontal axis, I_x, where x is perpendicular to the plane of the paper through point O. Noting that $I_x = I_{xc} + y_c^2 A$, where I_{xc} is the moment of inertia about the centroidal horizontal axis, (12) can be rewritten as

$$y_p\ h_c\ A = \sin\theta\ I_{xc} + \sin\theta\ y_c^2\ A \qquad (13).$$

But $h_c = \sin\theta\ y_c$ so we can finally write

$$y_p - y_c = \frac{I_{xc}}{y_c\ A} \qquad (14).$$

Equation (14) gives the position of the position where a single force, F, equivalent to the distributed hydrostatic force would be applied.