7.9 For low speed flow over a flat plate, one measure of the boundary layer is the resulting thickness, \( \delta \), at a given downstream location. The boundary layer thickness is a function of the free stream velocity, \( V_\infty \), fluid density and viscosity \( \rho \) and \( \mu \), and the distance from the leading edge, \( x \). Find the number of \( \pi \) terms for this relationship.

Given that
\[
\delta = f(V_\infty, \rho, \mu, x)
\]
where
\[
\delta = \frac{L}{\delta}, \quad V_\infty = \frac{L}{T}, \quad \rho = \frac{M}{L^3}, \quad \mu = \frac{M}{LT}, \quad \text{and} \quad x = \frac{L}{T}
\]
Thus, there are 5 variables and 3 basic dimensions (MLT) so that
\[
k - r = 5 - 3 = 2
\]
Hence, 2 \( \pi \) terms are needed: \( \eta = \phi(n_z) \)
7.13 Water sloshes back and forth in a tank as shown in Fig. P7.13. The frequency of sloshing, \( \omega \), is assumed to be a function of the acceleration of gravity, \( g \), the average depth of the water, \( h \), and the length of the tank, \( \ell \). Develop a suitable set of dimensionless parameters for this problem using \( g \) and \( \ell \) as repeating variables.

\[
\omega = f \left( g, h, \ell \right)
\]

\[
\omega = T^{-1}, \quad g = LT^{-2}, \quad h = L, \quad \ell = L
\]

From the pi theorem, \( 4 - 2 = 2 \) dimensionless parameters required. Use \( g \) and \( \ell \) as repeating variables, thus,

\[
\Pi_1 = \omega g^a \ell^b
\]

and \( (T^{-1})(LT^{-2})^a(L)^b = L^0 T^0 \)

so that

\[
a + b = 0
\]

\[
-1 - 2a = 0
\]

It follows that \( a = -\frac{1}{2}, b = \frac{1}{2} \), and therefore

\[
\Pi_1 = \omega \sqrt{\frac{g}{\ell}}
\]

Check dimensions:

\[
\omega \sqrt{\frac{g}{\ell}} = \frac{1}{T} \sqrt{\frac{L}{LT^{-2}}} = L^0 T^{-1} \quad \therefore \text{OK}
\]

For \( \Pi_2 \):

\[
\Pi_2 = h g^a \ell^b
\]

\( (LT^{-2})^a(L)^b = L^0 T^0 \)

\[
1 + a + b = 0
\]

\[
-2a = 0
\]

It follows that \( a = 0, b = -1 \), and therefore

\[
\Pi_2 = \frac{h}{\ell}
\]

and \( \Pi_2 \) is obviously dimensionless. Thus,

\[
\omega \sqrt{\frac{g}{\ell}} = \phi \left( \frac{h}{\ell} \right)
\]
7.14 Assume that the power, \( P \), required to drive a fan is a function of the fan diameter, \( D \), the fluid density, \( \rho \), the rotational speed, \( \omega \), and the flow rate, \( Q \). Use \( D, \omega, \) and \( \rho \) as repeating variables to determine a suitable set of pi terms.

\[
P = f (D, \omega, \rho, Q)
\]

\[
P = FLT^{-1} \quad D = L \quad \rho = FL^{-1} T^{-2} \quad \omega = T^{-1} \quad Q = L^3 T^{-1}
\]

From the pi theorem, \( 5 - 3 = 2 \) pi terms required. Use \( D, \omega, \) and \( \rho \) as repeating variables. Thus,

\[
\Pi_1 = D^a \omega^b \rho^c
\]

and

\[
FL^{-1}(L)^a(T^{-1})^b(FL^{-1} T^{-2})^c = P^0 L^0 T^0
\]

So that

\[
1 + c = 0 \quad (\text{for F})
\]

\[
1 + a - 4c = 0 \quad (\text{for L})
\]

\[
1 - b + 2c = 0 \quad (\text{for T})
\]

It follows that \( a = -5, \ b = -3, \ c = -1, \) and therefore

\[
\Pi_1 = \frac{P}{\rho D^5 \omega^3}
\]

Check dimensions using MLT system:

\[
\frac{P}{\rho D^5 \omega^3} = \frac{M L^2 T^{-3}}{(M L^{-3})(L)^2 (T^{-1})^3} = M^0 L^0 T^0 \quad : \text{OK}
\]

For \( \Pi_2 \):

\[
\Pi_2 = Q D^a \omega^b \rho^c
\]

\[
(L^3 T^{-1})(L)^a(T^{-1})^b(FL^{-1} T^{-2})^c = P^0 L^0 T^0
\]

\[
c = 0 \quad (\text{for F})
\]

\[
3 + a - 4c = 0 \quad (\text{for L})
\]

\[
-1 - b + 2c = 0 \quad (\text{for T})
\]

It follows that \( a = -3, \ b = -1, \ c = 0, \) and therefore

\[
\Pi_2 = \frac{Q}{D^3 \omega}
\]

Check dimensions using MLT system:

\[
\frac{Q}{D^3 \omega} = \frac{L^3 T^{-1}}{(L)^3 (T^{-1})} = M^0 L^0 T^0 \quad : \text{OK}
\]

Thus,

\[
\frac{P}{\rho D^5 \omega^3} = \phi \left( \frac{Q}{D^3 \omega} \right)
\]
7.15 Assume that the flowrate, $Q$, of a gas from a smokestack is a function of the density of the ambient air, $\rho_a$, the density of the gas, $\rho_g$, within the stack, the acceleration of gravity, $g$, and the height and diameter of the stack, $h$ and $d$, respectively. Use $\rho_a$, $d$, and $g$ as repeating variables to develop a set of pi terms that could be used to describe this problem.

$$Q = f(\rho_a, \rho_g, g, h, d)$$

$$Q \equiv L^3 T^{-1} \quad \rho_a \equiv ML^{-3} \quad \rho_g \equiv ML^{-3} \quad g \equiv LT^{-2} \quad h \equiv L \quad d \equiv L$$

From the pi theorem, $6-3=3$ pi terms required. Use $\rho_a$, $d$, and $g$ as repeating variables. Thus,

$$\Pi_1 = \frac{Q}{\rho_a} d^b g^c$$

and

$$(L^3 T^{-1})(ML^{-3})^a(L)^b((LT^{-2})^c = M^0 L^0 T^0$$

so that

$$a = 0 \quad \text{ (for } M)$$
$$3 - 3a + b + c = 0 \quad \text{ (for } L)$$
$$-1 - 2c = 0 \quad \text{ (for } T)$$

It follows that $a = 0$, $b = -\frac{5}{2}$, $c = -\frac{1}{2}$, and therefore

$$\Pi_1 = \frac{Q}{d^{5/2} g^{1/2}}$$

Check dimensions using FLT system:

$$\frac{Q}{d^{5/2} g^{1/2}} \equiv \frac{L^3 T^{-1}}{(L)^{5/2} (LT^{-2})^{1/2}} = M^0 L^0 T^0 \quad \therefore \text{ OK}$$
For $\Pi_2$:

$$\Pi_2 = \frac{\rho^a}{\rho_0^a} \frac{d^b}{d_0^b} \frac{q^c}{q_0^c}$$

$$(ML^{-3})(ML^{-3})^a(L)^b(LT^{-2})^c = M^a L^b T^c$$

$$1 + a = 0 \quad \text{(for M)}$$
$$-3 - 3a + b + c = 0 \quad \text{(for L)}$$
$$-2c = 0 \quad \text{(for T)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{\rho}{\rho_0}$$

which is obviously dimensionless.

For $\Pi_3$:

$$\Pi_3 = \frac{h}{\rho_0^a}$$

$$(L)(ML^{-3})^a(L)^b(LT^{-2})^c = M^a L^b T^c$$

$$a = 0 \quad \text{(for M)}$$
$$1 - 3a + b + c = 0 \quad \text{(for L)}$$
$$-2c = 0 \quad \text{(for T)}$$

It follows that $a = 0$, $b = -1$, $c = 0$, and therefore

$$\Pi_3 = \frac{h}{d}$$

which is obviously dimensionless.

Thus,

$$\frac{\Phi}{d^{3/2} q^{1/2}} = \Phi \left( \frac{\rho}{\rho_0}, \frac{h}{d} \right)$$
7. The pressure rise, $\Delta p$, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where $D$ is the impeller diameter, $\rho$ the fluid density, $\omega$ the rotational speed, and $Q$ the flowrate. Determine a suitable set of dimensionless parameters.

$$\Delta p = FL^{-2} D = L \quad \rho = FL^{-4} T^2 \quad \omega = T^{-1} \quad Q = L^3 T^{-1}$$

From the pi theorem, $5-3 = 2$ pi terms required. Use $D, \rho$, and $\omega$ as repeating variables. Thus,

$$\Pi_1 = \frac{\Delta p}{D^a \rho^b \omega^c}$$

and

$$(FL^{-2})(L)^a (FL^{-4} T^2)^b (T^{-1})^c = 100 L^0 T^0$$

so that

$$1 + b = 0 \quad \text{(for F)}$$

$$2 + a - 4b = 0 \quad \text{(for L)}$$

$$2b - c = 0 \quad \text{(for T)}$$

It follows that $a = -2, b = -1, c = -2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2 \rho \omega^2} = \frac{M L^{-1} T^{-2}}{(L)^2 (M L^{-3} T^{-1})^2} = M_0 L_0 T_0 \quad \therefore \text{OK}$$

For $\Pi_2$:

$$\Pi_2 = \frac{\phi}{D^a \rho^b \omega^c}$$

$$(L^3 T^{-1})(L)^a (FL^{-4} T^2)^b (T^{-1})^c = 100 L^0 T^0$$

$$b = 0 \quad \text{(for F)}$$

$$3a + 4b = 0 \quad \text{(for L)}$$

$$-1 + 2b - c = 0 \quad \text{(for T)}$$

It follows that $a = -3, b = 0, c = -1$, and therefore

$$\Pi_2 = \frac{\phi}{D^3 \omega}$$

Check dimensions using MLT system:

$$\frac{\phi}{D^3 \omega} = \frac{L^3 T^{-1}}{(L)^3 (T^{-1})^1} = M_0 L_0 T_0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\Delta p}{D^2 \rho \omega^2} = \phi \left( \frac{\phi}{D^3 \omega} \right)$$
7.38 To test the aerodynamics of a new prototype automobile, a scale model will be tested in a wind tunnel. For dynamic similarity, it will be required to match Reynolds number between model and prototype. Assuming that you will be testing a one-tenth-scale model and both model and prototype will be exposed to standard air pressure, will it be better for the wind tunnel air to be colder or hotter than standard sea-level air temperature of 15 °C? Why?

Let \( R_e \) denote model and prototype, respectively. Thus, \( R_e = R_{ep} \), or

\[
\left( \frac{V}{
u} \right)_m = \left( \frac{V}{
u} \right)_p, \quad \text{or}
\]

\[
V_m = \frac{V_m}{\nu_p} \frac{\nu_p}{\nu_m} V_p = 10 \frac{V_m}{\nu_p} V_p \quad \text{since} \quad \frac{\nu_m}{\nu_p} = \frac{1}{10} \frac{\nu_m}{\nu_p}
\]

If the wind tunnel air is at standard sea-level conditions, then \( V_m = V_p \) and

\[
V_m = 10 V_p. \quad \text{Hence, if} \quad V_p = 55 \text{ mph, then} \quad V_m = 550 \text{ mph, which is too large for simple tests. For one this, at 550 mph compressibility effects become important. At 55 mph they are not. Assume the test are conducted with} \quad \frac{V_m}{\nu_p} < 1 \quad \text{so that more realistic wind tunnel velocities are prescribed. From the data in Table B.4, at} \quad T = 15 \text{ °C,} \quad \nu_p = 1.47 \times 10^{-5} \text{ m}^2/\text{s and the data shown below are obtained.}
\]

![Graph](image)

For \( \frac{V_m}{\nu_p} < 1 \) it follows that \( T = 15 \text{ °C} \). Hence, it would be better to have a cold wind tunnel. However, even with \( T = -40 \text{ °C} \), which gives \( \frac{V_m}{\nu_p} = 0.707 \), the \( V_p = 55 \text{ mph} \) would require \( V_m = 10(0.707)55 \text{ mph} = 389 \text{ mph} \).
7.39 You are to conduct wind tunnel testing of a new football design that has a smaller lace height than previous designs (see Videos V6.1 and V6.2). It is known that you will need to maintain Re and St similarity for the testing. Based on standard college quarterbacks, the prototype parameters are set at $V = 40\text{ mph}$ and $\omega = 300\text{ rpm}$. The prototype football has a 7-in. diameter. Due to instrumentation required to measure pressure and shear stress on the surface of the football, the model will require a length scale of 2:1 (the model will be larger than the prototype). Determine the required model freestream velocity and model angular velocity.

Let $(\cdot)_m$ and $(\cdot)_p$ denote the model and prototype, respectively.

For Reynolds number similarity, $Re_m = Re_p$, or

\[
\frac{V_m D_m}{V_m} = \frac{V_p D_p}{V_p}, \text{ so that with } V_m = V_p \text{ (i.e., same air properties)}
\]

\[
V_m = \frac{D_p V_p}{D_m} = \left(\frac{1}{2}\right) (40\text{ mph}) = 20\text{ mph}, \text{ since } D_m = 2D_p.
\]

Thus,

\[
V_m = 20\frac{\text{mi}}{\text{hr}} \left(\frac{5280\text{ ft}}{\text{mi}}\right) \left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 29.3\frac{\text{ft}}{\text{s}}
\]

For Strouhal number similarity, $St_m = St_p$, or

\[
\frac{\omega_m D_m}{V_m} = \frac{\omega_p D_p}{V_p}, \text{ where } \omega_p = 300\text{ rpm}
\]

Hence,

\[
\omega_m = \frac{V_m}{V_p} \frac{D_p}{D_m} \omega_p = \left(\frac{20\text{ mph}}{40\text{ mph}}\right) \left(\frac{1}{2}\right) (300\text{ rpm}) = 75\text{ rpm}
\]
7.48 The lift and drag developed on a hydrofoil are to be determined through wind tunnel tests using standard air. If full scale tests are to be run, what is the required wind tunnel velocity corresponding to a hydrofoil velocity in seawater of 15 mph? Assume Reynolds number similarity is required.

For Reynolds number similarity,

\[ \frac{V_m l_m}{V_m} \frac{l}{l_m} = \frac{V}{V} \]

where \( l \) is some characteristic length of the hydrofoil. Thus,

\[ V_m = \frac{V_m}{V} \frac{l}{l_m} V \]

and with \( l/l_m = 1 \) (full scale test),

\[ V_m = \frac{1.57 \times 10^{-4} \frac{ft^2}{s}}{1.26 \times 10^{-5} \frac{ft^2}{s}} (15 \text{ mph}) \]

= 187 mph
7.4 A 1/50 scale model is to be used in a towing tank to study the water motion near the bottom of a shallow channel as a large barge passes over. (See Video V7.16) Assume that the model is operated in accordance with the Froude number criteria for dynamic similarity. The prototype barge moves at a typical speed of 15 knots. (a) At what speed (in ft/s) should the model be towed? (b) Near the bottom of the model channel a small particle is found to move 0.15 ft in one second so that the fluid velocity at that point is approximately 0.15 ft/s. Determine the velocity at the corresponding point in the prototype channel.

(a) For Froude number similarity

\[ \frac{V_m}{\sqrt{g_m l_m^2}} = \frac{V}{\sqrt{g l}} \]

where \( l \) is some characteristic length, and with \( g_m = g \)

\[ \frac{V_m}{V} = \sqrt{\frac{l_m}{l}} \] (1)

Thus,

\[ V_m = \sqrt{\frac{1}{50}} \ (15 \text{ knots}) = 2.12 \text{ knots} \]

From Table A.1  

1 knot = \( 0.514 \, \text{m/s} \) \( \times \) \( 3.28 \, \frac{\text{ft}}{\text{m}} \) = 1.69 \( \frac{\text{ft}}{\text{s}} \)

So that

\[ V_m = (2.12 \text{ knots}) \left( 1.69 \, \frac{\text{ft}}{\text{knot}} \right) = 3.58 \, \frac{\text{ft}}{\text{s}} \]

(b) Since from Eq. (1)

\[ \frac{V_m}{V} = \sqrt{\frac{l_m}{l}} = \sqrt{\frac{1}{50}} \]

So that

\[ V = \sqrt{50} \ (0.15 \, \frac{\text{ft}}{\text{s}}) = 1.06 \, \frac{\text{ft}}{\text{s}} \]
7.60 As shown in Fig. P7.60, a “noisemaker” B is towed behind a minesweeper A to set off enemy acoustic mines such as at C. The drag force of the noisemaker is to be studied in a water tunnel at a \( \frac{1}{3} \) scale model (model \( \frac{1}{3} \) the size of the prototype). The drag force is assumed to be a function of the speed of the ship, the density and viscosity of the fluid, and the diameter of the noisemaker. (a) If the prototype towing speed in 3 m/s, determine the water velocity in the tunnel for the model tests. (b) If the model tests of part (a) produced a model drag of 900 N, determine the drag expected on the prototype.

\[
D = f(V, \rho, \mu, D), \quad \text{where} \quad D = F = \frac{M}{\frac{V^2}{2}}, \quad V = \frac{L}{T}, \quad \rho = \frac{M}{L^3}, \quad \mu = \frac{M}{L^2}, \quad \text{and} \quad D = L
\]

Thus, \( k = 5 - 3 = 2 \) so that \( \eta_1 = \eta(\eta_2) \),

where by inspection there are the ingredients for a Reynolds number, \( Re = \frac{\eta V D}{\mu} \), and a drag coefficient, \( C_D = \frac{D}{\frac{1}{2} \rho V^2 D^2} \).

Hence,

\[ C_D = \Phi(Re) \]

For similarity, \( Re_m = Re \), or

\[ \frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu} \]

so that with \( \rho_m = \rho \) and \( \mu_m = \mu \),

\[ V_m D_m = V D \]

or with \( D_m = \frac{1}{4} D \),

\[ V_m = \frac{D}{D_m} V = 4 V = 4 \left( \frac{3}{5} \right) = 12 \frac{m}{s} \]

(b) With \( Re_m = Re \) it follows that \( C_{D_m} = C_D \), or

\[ \frac{\rho_m V_m^2 D_m^2}{\frac{1}{2} \rho V^2 D^2} = \frac{\rho V D}{\frac{1}{2} \rho V^2 D^2} \]

Thus, since \( \rho_m = \rho \),

\[ \frac{D_m}{V_m D_m^2} = \frac{\rho V D}{V^2 D^2} \]

\[ D = \left( \frac{V}{V_m} \right)^2 \left( \frac{D}{D_m} \right)^2 D_m = \left( \frac{3 m/s}{12 m/s} \right)^2 (4)^2 (900 N) = 900 N \]

Note: The prototype has the same drag as the model.